REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE April 30, 2002	3. REPORT TYPE AND DATES COVERED Final: May 1, 1999 – April 30, 2002

4. TITLE AND SUBTITLE
Analysis of Bivariate and Trivariate Macro-Elements for Surface Fitting

5. FUNDING NUMBERS
DAAD-19-99-1-0160

6. AUTHOR(S)
Larry L. Schumaker

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Vanderbilt University
Nashville, TN 37240

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
39856.21-MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The work performed under this grant has focused on developing specific spline tools (and the necessary underlying theory). The results can be of use for a variety of applied problems, including for example a) scattered data fitting of very large data sets (such as arise in digital terrain modelling, geosciences, meteorology, etc.), and b) numerical solution of boundary-value problems by finite-element methods. The work has resulted in a total of 16 research papers.

14. SUBJECT TERMS
Splines

15. NUMBER OF PAGES
3

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

Enclosure 1

FINAL REPORT: DAAD 19-99-1-0160

Larry L. Schumaker, Vanderbilt University

§1. Foreword

The work performed under this grant has focused on developing specific spline tools (and the necessary underlying theory). The results can be of use for a variety of applied problems, including for example a) scattered data fitting of very large data sets (such as arise in digital terrain modelling, geosciences, meteorology, etc.), and b) numerical solution of boundary-value problems by finite-element methods. The work has resulted in a total of 16 research papers (see list below).

§2. Summary of Results

A) Macro-Elements. Macro-elements based on piecewise polynomials on triangulations are useful for both scattered data fitting and for solving boundary value problems. In recent years the classical elements used by engineers have been generalized in various papers to C^r smoothness. Most of these higher-smoothness elements were not, however, optimal with respect to various properties such as degree, number of degrees of freedom, approximation power, etc. The purpose of the papers [3,5,9,13] was to develop optimal sequences of smooth macro elements on both the Clough-Tocher and Powell-Sabin triangle splits.

It is also useful to have macro-elements defined on triangulated quadrangulations similar to the well-known FVS elements. In [6] we develop a complete family of such elements for all smoothness. Our work also produced analogs of all of these elements for spaces of spherical splines on spherical triangulations.

B) Spline Fitting. Many spline fitting methods involve minimizing some combination of energy and goodness of fit over an appropriate prescribed spline space. While existence and uniqueness of solutions of such minimization problems has generally been well-understood (along with practical computational methods), little was known about error bounds for the methods. In [7] we establish general results on projections into spline spaces, and in [14] apply these results to get specific error bounds for minimal energy interpolation methods.

C) A Spherical Multi-resolution Method. Problems in geophysics and meteorology generally involve functions defined on the earth, and can usually be transformed to problems on the sphere. Much is known about multi-resolution (wavelet) methods for planar functions, but much less was known for the sphere. In [1] we develop a spline-based multi-resolution

method. The efficiency and power of the method was demonstrated by fitting ETOPO-5 terrain data.

D) Interpolation Methods. The most effective methods for interpolating large sets of scattered data are local methods (such as the macro-element methods mentioned above). In [8] we showed how recent alternative C^1 methods based on quartic splines could be stabilized to insure optimal order error bounds.

In [11,12] we investigated the problem of creating spline spaces (based on triangulated quadrangulations) and associated point sets which can be used to perform Lagrange interpolation.

E) Spline Theory. One of the most important problems in spline theory is to construct *stable local bases*. What is needed is an algorithm which will produce a basis for a given spline space so that the size of the basis elements and also the size of their support sets are bounded by constants depending only on the smallest angle in the triangulation. Constructing such algorithms is a delicate matter, and is done for large classes of splines and supersplines in [4,10]. Similar algorithms producing *locally linearly independent bases* were obtained in [2].

The paper [15] generalizes classical work on upper and lower bounds for dimensions of spline spaces to much wider classes of super-spline spaces. As an example of the applicability of the results, the bounds are used to analyze a certain C^2 macro-element on a double Clough-Tocher split.

Finally, in [16] we examine how well smooth functions defined on the sphere can be approximated by certain classes of spherical splines defined on spherical triangulations.

§3. Publications

(a) Journals

1. A multiresolution tensor spline method for fitting functions on the sphere, with T. Lyche, SIAM J. Scient. Computing **22** (2000), 724–746.
2. Locally linearly independent bases for bivariate polynomial spline spaces, with Oleg Davydov, Advances in Comp. Math. **13** (2000), 355–373.
3. Macro-elements and stable local bases for spaces of splines on Clough-Tocher triangulations, with M.-J. Lai, Numer. Math. **88** (2001), 105–119.
4. On stable local bases for bivariate polynomial spline spaces, with Oleg Davydov, Constr. Approx. **18** (2001), 87–116.
5. Smooth macro-elements based on Powell-Sabin triangle splits, with P. Alfeld, Advances in Comp. Math. **16** (2002), 29–46

6. Quadrilateral macro-elements, with Ming-Jun Lai, SIAM J. Math. Anal. **33** (2002), 1107–1116.
7. Bounds on projections onto bivariate polynomial spline spaces with stable bases, with Manfred von Golitschek, Constr. Approx. **18** (2002), 251–254.
8. Stable approximation and interpolation with C^1 quartic bivariate splines, with Oleg Davydov, SIAM J. Numer. Anal. **39** (2002), 1732–1748.
9. Smooth macro-elements based on Clough-Tocher triangle splits, with P. Alfeld, Numer. Math. **90** (2002), 597–616.

(b) Proceedings

10. Stable local nodal bases for C^1 bivariate polynomial splines, with Oleg Davydov, in *Curve and Surface Design: Saint-Malo 99*, P.-J. Laurent, P. Sablonnière, and L. L. Schumaker (eds.), Vanderbilt University Press, Nashville, 2000, 171–180.
11. Local Lagrange interpolation by bivariate C^1 cubic splines, with G. Nürnberger and F. Zeilfelder, *Mathematical Methods for Curves and Surfaces III, Oslo, 2000*, T. Lyche and L. L. Schumaker (eds.), Vanderbilt University Press, 2001, 393–404.
12. Local Lagrange interpolation by C^1 cubic splines on triangulations of separable quadrangulations, with G. Nürnberger and F. Zeilfelder, in *Approximation Theory X: Splines, Wavelets, and Applications*, Charles K. Chui and Larry L. Schumaker (eds.), Vanderbilt University Press, Nashville, 2002, 405–424.

(c) To Appear

13. Macro-elements and stable local bases for spaces of splines on Powell-Sabin triangulations, with M.-J. Lai, Math. Comp., to appear.
14. Error bounds for minimal energy bivariate polynomial splines, with M.-J. Lai and Manfred von Golitschek, Numer. Math., to appear.

(d) Submitted

15. Upper and lower bounds on the dimension of superspline spaces, with Peter Alfeld, Constr. Approx., submitted.
16. On the approximation order of splines on spherical triangulations, with Mike Neamtu, Advances in Comp. Math., submitted.

§4. Scientific Personnel

Three graduate students participated in the research. Tanya Morton earned a PhD in mathematics from Vanderbilt in spring of 2000. Vera Rayevskaya is currently working on her PhD thesis. Tatiana Sorokina is working on PhD qualifying exams.